



Developing Mathematical Thinking:
Changing Teachers' Knowledge and Instruction

RESEARCH OVERVIEW
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DEVELOPING MATHEMATICAL THINKING: CHANGING TEACHERS' KNOWLEDGE AND INSTRUCTION

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Introduction

In mathematics education, the journey toward effective teaching begins with a deep understanding of what students know and need to learn. This understanding and appropriate challenges and support form the cornerstone of effective mathematics pedagogy. The research underscores the importance of ensuring that students not only grasp mathematical concepts but also learn to think critically, solve problems, and view mathematics as an interconnected web of knowledge rather than isolated topics.

Despite national recommendations emphasizing the importance of fostering communication, reasoning, and problem-solving skills in students, the reality in many classrooms falls short of these ideals. Teachers often lack the rich mathematical experiences necessary to develop the multifaceted knowledge required for effective teaching. Furthermore, the gap between the envisioned standards and actual classroom practice persists, indicating a pressing need for rigorous professional development initiatives.

Transforming Knowledge

Addressing these challenges necessitates a multifaceted approach to transforming teachers' knowledge of mathematics, pedagogy, and student learning. Ball and colleagues' framework for teacher knowledge provides a valuable guide, emphasizing the importance of content knowledge, pedagogical content knowledge, and knowledge of student thinking. For elementary teachers, a shift toward deeper conceptual understanding is crucial, while secondary teachers must develop specialized knowledge grounded in the acts of teaching (Ball, Hill, & Bass, 2005).

Teachers' knowledge must extend beyond mere procedural understanding to encompass the ability to analyze multiple solution strategies, anticipate student misconceptions, and select appropriate instructional practices. Such knowledge forms the foundation for creating intellectually rich learning environments aligned with the principles of teaching for understanding.

Transforming Practice

While knowledge forms the bedrock of effective teaching, cultivating a professional culture through collaboration and inquiry is equally vital. Encouraging teachers to adopt an inquiry stance fosters ongoing reflection and critical examination of instructional practices. Teachers can refine their instructional approaches and enhance student learning outcomes by engaging in productive discussions and collaborations.

A theoretical framework rooted in hypothetical learning trajectories (HLT) provides a roadmap for guiding teachers' professional development. HLTs offer a dynamic vision of students' mathematical progression, guiding teachers' task selection and instructional sequences. By aligning professional development activities with HLTs, teachers gain insights into students' evolving understanding and can tailor their instructional practices accordingly.

A Model for Teaching for Understanding: DMT Instructional Theory

Central to transforming teaching practice is the adoption of a model rooted in guided reinvention and mathematizing. Guided reinvention empowers students to develop informal problem-solving strategies, gradually transitioning towards more formal mathematical reasoning. Through mathematizing, students deepen their understanding by abstracting mathematical concepts and generalizing solution strategies.

Teachers play a pivotal role in facilitating this process, fostering a classroom culture conducive to mathematical exploration and discourse. Learning tasks should provoke students' curiosity and promote mathematical reasoning while teachers guide students toward deeper conceptual understanding. By leveraging mathematical tools and promoting equitable access to learning, teachers create inclusive learning environments that empower all students to engage meaningfully with mathematics.

Results

The primary aim of this investigation was to assess the efficacy of our professional development program in enhancing teachers' mathematics knowledge and classroom practice. We begin by presenting descriptive statistics for content knowledge and observational data of teaching practice. Subsequently, we explore the relationship between knowledge and teaching practice.

Changes in Teachers' Content Knowledge

The pretest scores on the knowledge inventories indicate limited mathematical knowledge across the three domains. These findings align with previous research.

The knowledge inventory was re-administered in the spring, six months after the initial training. Posttest scores were significantly higher than pretest scores for all domains (number and operations, measurement and geometry, and probability and statistics), indicating a significant increase in teachers' overall mathematical knowledge.

Observation of Teachers' Practice

We computed an average score across the items, making up the six features of the teacher observation instrument. These scores reflect observable behaviors related to classroom tasks, the teacher's role, the social culture of the classroom, mathematical tools as learning supports, equity and accessibility, and classroom discourse.

Teacher Content Knowledge in Relation to Instruction

We used Pearson correlations to examine the relationship between teachers' content knowledge and their instructional practices. Interestingly, initial content knowledge (pretest score) was not correlated with instructional practices, but significant correlations were found between teachers' content knowledge gains and their instructional practices post-professional development.

Furthermore, the number of years a teacher had taught was not significantly related to content knowledge or instructional practice scores. However, a slight inverse relation existed between years of experience and instructional practice scores, suggesting that more experienced teachers may exhibit less alignment with teaching for understanding.

Discussion, Conclusions, and Implications

This study highlights the positive impact of professional development on teachers' mathematical knowledge and instructional practices. The findings suggest that engaging in reform-oriented mathematics teaching leads to significant changes in both knowledge and practice.

Teachers who actively implemented reform strategies in their classrooms demonstrated greater content knowledge gains. This underscores the importance of practical application in professional development initiatives.

Moreover, the study emphasizes the necessity of aligning professional development with teaching for understanding principles. Teachers who embraced these principles exhibited more significant gains in both content knowledge and instructional practices.

The findings also indicate that content knowledge alone may not suffice to improve instructional practices. Instead, efforts should focus on providing teachers with the tools and strategies necessary for implementing effective teaching methodologies.

Overall, this study underscores the importance of ongoing, reform-oriented professional development in enhancing teachers' mathematical knowledge and instructional practices, ultimately benefiting student learning outcomes.



For more information

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