DMTI
Math Success

Developing Mathematical Thinking:
Focusing on Mathematical Structure

RESEARCH OVERVIEW
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DEVELOPING MATHEMATICAL THINKING INSTITUTE

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BUILDING STRONG THINKERS: UNLEASHING THE POWER OF MATHEMATICAL STRUCTURE!

(RESEARCH OVERVIEW)

Introduction

Mathematical structure is fundamental to effective teaching and comprehension in mathematics education. Educators can significantly enhance students' abilities to solve problems, develop conceptual understanding, and apply mathematical reasoning across various contexts by understanding and leveraging underlying structures- such as patterns, relationships, and fundamental concepts. This research overview integrates insights from the broader research on mathematical structure, emphasizing structural language's importance in describing fundamental ideas in mathematics.

The Importance of Mathematical Structure

Mathematical structure refers to the consistent, foundational components that recur across topics and grade levels. These include essential ideas like unit, compose, decompose, iterate, partition, and equal. Understanding and using these terms is critical to students' mathematical development, as they help build a coherent framework for learning. For example, recognizing that the number 28 is composed of two units of 10 and eight units of 1 fosters a deep understanding of place value. Similarly, partitioning 28 into 4 units of 7 helps them grasp the concept of division.

The language of structure—unit, compose, decompose, iterate, partition, and equal—should be embedded in classroom instruction to support students in making connections across mathematical topics (Brendefur et al., 2021). For example, composing and decomposing numbers are foundational skills that extend from early arithmetic to more complex topics such as algebra, where students must decompose expressions into simpler forms.

Recognizing and applying structural language also aligns with the work of Inglis and Foster (2018), who argue that understanding the coherence of mathematical ideas across the curriculum is essential for students' long-term success. By focusing on the structural elements common across different grade levels, educators can help students build a solid mathematical foundation to tackle more complex problems as they advance in their education.

Structure in the Common Core State Standards

The Common Core State Standards (CCSS) explicitly emphasize the importance of students developing the ability to "look for and make use of structure" (Mathematical Practice Standard 7). This practice encourages students to discern patterns, break down complex problems into simpler components, and recognize the connections between different mathematical concepts. For example, young students might identify that adding 3 + 7 is the same as 7 + 3, demonstrating the concept of equivalence. On the other hand, older students might decompose multiplication problems using the distributive property, such as recognizing that 7×8 is the same as (7×5)+(7×3), which shows how iteration of a unit can simplify problem-solving.

The CCSS also highlights the significance of structural language when tackling more advanced mathematical problems. For example, students may partition geometric shapes to find areas, decompose algebraic expressions into more manageable parts, or understand equivalent expressions. By encouraging students to think about problems in terms of composition and decomposition, teachers can help them approach complex problems confidently and flexibly.

Moreover, focusing on structural components such as unit, compose, decompose, iterate, partition, and equal provides students with a consistent framework they can apply across various mathematical topics. This coherence allows them to see the connections between seemingly disparate areas of mathematics, such as geometry and algebra, reinforcing their understanding of how mathematical ideas interrelate.

Classroom Implications and Teaching Strategies

Emphasizing mathematical structure in the classroom significantly affects how mathematics should be taught. The DMTI framework advocates embedding the language of structural components—unit, compose, decompose, iterate, partition, and equal—into every lesson. By using these terms regularly, teachers can help students articulate their understanding of mathematical concepts and recognize the structural connections between different topics.

One effective teaching strategy that supports this approach is concept mapping. Concept mapping encourages students to visualize the relationships between different mathematical ideas, making it easier to see how various concepts are composed of smaller units or can be decomposed into simpler parts. This strategy aligns with the CCSS's emphasis on structure and fosters deeper mathematical proficiency by helping students recognize patterns and relationships across topics.

The Role of Professional Development

Professional development is crucial for successfully integrating structural language into mathematics instruction. Educators must be well-versed in structural language and understand how these concepts apply across grade levels. Professional development programs, such as those offered by the DMTI, aim to equip teachers with the tools to foster structural awareness in their students.

Research on Mathematical Structure

The literature on mathematical structure emphasizes its crucial role in enhancing students' understanding and problem-solving abilities. Mulligan et al. (2009) argue that an awareness of mathematical patterns and structures is essential for students to engage effectively with mathematical concepts. Their research highlights the importance of teaching students to compose and decompose mathematical problems, allowing them to break down complex tasks into simpler, more manageable parts.

Wittmann (2020) expands on this idea by helping students see the logical patterns that form the foundation of mathematics. By identifying the underlying structure, students learn to break down complex problems into simpler parts. This makes them more effective problem solvers and better equipped to tackle unfamiliar math challenges. By teaching with this focus on structure, we are giving students the tools to understand math at a deeper level and use that understanding in various contexts.

Implications for Curriculum Development

The emphasis on mathematical structure has significant implications for curriculum development. Inglis and Foster (2018) note that curricula that highlight the interconnectedness of mathematical ideas help students develop a more coherent understanding of the subject. By integrating the structural language of mathematics—unit, compose, decompose, iterate, partition, and equal—into curriculum design, educators can ensure that students are equipped with the tools they need to approach mathematics confidently and flexibly.

Furthermore, research suggests that incorporating real-world problems into the curriculum can help students meaningfully apply their understanding of mathematical structures. Students who recognize how composing, decomposing, partitioning, and iteration apply to real-world situations are better prepared to transfer their mathematical knowledge to new contexts.

Conclusion

Recognizing and utilizing mathematical structure is essential for developing mathematical proficiency. The research on mathematics education emphasizes the importance of structural language in helping students develop a deeper understanding of mathematics. Educators can foster a classroom environment where students are encouraged to explore the relationships between mathematical ideas through concept mapping, self-regulated learning, and professional development. By emphasizing structural terms like unit, compose, decompose, iterate, partition, and equal, both in curriculum design and classroom instruction, educators can equip students with the tools they need to become flexible, independent problem solvers prepared for academic and real-world mathematics challenges.

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