



Developing Mathematical Thinking:
Strategies and Models

RESEARCH OVERVIEW

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THE IMPORTANCE OF MODELS IN MATHEMATICS EDUCATION

(RESEARCH OVERVIEW)

Introduction

In mathematics education, fostering a learning environment that encourages a variety of problem-solving strategies and emphasizes the structural foundations of mathematical concepts is crucial for student success. One key instructional element is using mathematical models to help students bridge their informal understandings with formal, symbolic mathematical reasoning. Encouraging students to use models, particularly iconic representations, is vital in developing conceptual and procedural knowledge. This research overview explores how modeling enhances student learning by progressing from intuitive representations to more formalized mathematical reasoning, focusing on the importance of iconic models in building a deeper understanding of mathematical concepts.

The Role of Models in Mathematical Thinking

Modeling is a powerful tool for nurturing mathematical thinking because it helps students move from concrete experiences to abstract reasoning. According to Romberg and Kaput (1999), when students first encounter mathematical problems, they naturally rely on informal strategies based on their real-world experiences. The modeling process allows these initial intuitive approaches to serve as scaffolding for solving more complex, related problems. Through modeling, students solve a specific problem and develop general strategies that can be applied across different mathematical contexts.

Gravemeijer and van Galen (2003) argue that modeling real-world situations is foundational for understanding mathematical structures. This process often begins with students using informal, tangible representations, which evolve into more formal mathematical reasoning as they progress. Cobb (2000) describes this as a shift in classroom practice, where students' informal activities, such as using objects or drawings, are eventually formalized into symbolic mathematical reasoning. The key to this transformation lies in how well students can transition between different forms of representation: enactive, iconic, and symbolic models (Bruner, 1964).

The Progression of Mathematical Models

A critical component of effective mathematics instruction is the concept of **progressive formalization**, which guides students through the stages of representation. As students work through mathematical problems, they begin with **enactive models**—physical representations or manipulatives that help them visualize the problem. From there,

students move on to **iconic models**, which involve visual or pictorial representations, such as number lines, bar models, diagrams, or graphs, that symbolize the relationships in the problem. Finally, they transition to **symbolic models**, which use formal mathematical notation and equations to represent abstract concepts (Bruner, 1964).

The transition from iconic to symbolic models is particularly important because it helps students visualize and understand abstract mathematical concepts without losing the connection to real-world problems. In many curricula, students are often asked to solve problems using multiple methods, but these methods may not always lead to the progressive formalization needed for deep understanding. Iconic models, such as area models for multiplication or division, serve as a critical bridge between concrete and abstract reasoning, allowing students to visualize the relationships between numbers and operations before transitioning to formal symbols (Leinwand & Ginsburg, 2007).

Iconic Models and Their Importance

Iconic models play a unique role in mathematics education by offering visual representations that build spatial reasoning and make abstract concepts more accessible. For example, the **area model** is a powerful iconic representation used in teaching multiplication and division. When students are presented with a contextualized problem, such as determining the number of tiles needed to cover a floor, they can use an area model to visualize the relationships between length, width, and area. This iconic representation helps students see multiplication as a repeated addition and a relationship between dimensions, preparing them for more formal mathematical concepts such as algebra (Watanabe, 2015).

The strength of iconic models lies in their ability to illuminate different aspects of mathematical relationships. Unlike abstract symbolic representations, which can be difficult for students to grasp, iconic models make the problem tangible and concrete. Students can manipulate the models, explore different problem-solving strategies, and visually see the consequences of their actions. This tactile and visual exploration deepens their conceptual understanding and supports the transition to more abstract forms of reasoning (Bruner, 1964).

For instance, using a **number line** as an iconic model for fractions allows students to visualize the relative size of different fractions, helping them understand concepts such as equivalence and comparison. Similarly, **bar models** can represent proportions, ratios, or algebraic relationships. These iconic models provide a clear visual framework for understanding the underlying structure of mathematical problems, and they encourage students to explore multiple solution strategies.

Modeling in Curriculum Design

Integrating modeling into mathematics curricula has fostered deeper student engagement and understanding. However, educators must select contexts and tasks that naturally lead students from informal models to more formal, mathematically robust representations. For example, when teaching multiplication, students may begin by solving problems about grouping objects. These problems encourage using iconic models, such as creating linear bar models and then area models to represent multiplication, before transitioning to symbolic equations (Leinwand & Ginsburg, 2007).

Curricula that prioritize the progression from enactive to iconic to symbolic models help students build a solid foundation for understanding more advanced mathematical concepts. For example, suppose an educator aims for students to use the area model as an iconic representation. In that case, they might introduce problems involving geometric concepts, such as covering flat spaces with tiles or using gridlines on a map to calculate distances. These activities make math more tangible and foster the logical connections necessary for students to develop more formal mathematical reasoning (Watanabe, 2015).

Additionally, students' engagement with different models enhances their ability to communicate and justify their mathematical thinking. When asked to explain how they arrived at a solution using an iconic model, they must articulate the mathematical relationships they observe, which promotes a deeper conceptual understanding. This process also aligns with socio-mathematical **norms**, where students learn to evaluate the efficiency and effectiveness of different models and strategies through classroom discussion and peer feedback.

The Cognitive Benefits of Modeling

From a cognitive psychology perspective, using models in mathematics education helps bridge the gap between procedural and conceptual knowledge. Research by Gilmore et al. (2009) suggests that procedural fluency and conceptual understanding are interconnected, with advancements in one area reinforcing the other. The iterative development of models provides students with opportunities to build conceptual knowledge—by visualizing and manipulating the mathematical structures underlying the problems they solve—and procedural skills—through repeated practice.

Bruner's (1964) theory of representation emphasizes the importance of guiding students through the different representational forms—enactive, iconic, and symbolic—without imposing abrupt transitions. The gradual transition from one form of representation to another enables students to develop a deeper, more integrated understanding of mathematical concepts, reducing the cognitive load associated with learning new material. This approach allows students to internalize mathematical concepts more effectively, making them better prepared to tackle more complex problems in the future.

Conclusion

In conclusion, mathematical modeling is a critical framework for helping students develop a deeper understanding of mathematics by progressing through enactive, iconic, and symbolic representations. Iconic models, in particular, are essential for bridging the gap between students' informal understandings and the abstract formalism of mathematical reasoning. Educators can foster environments where students are encouraged to explore, innovate, and deepen their understanding of mathematical structures by emphasizing using models in mathematics instruction. This progressive formalization supports conceptual knowledge and procedural fluency and prepares students to thrive in mathematics and beyond.

Integrating modeling into curricula and thoughtfully selecting tasks that support the progression from informal to formal reasoning empowers students to recognize the diverse problem-solving methods. It encourages them to develop their unique mathematical insights. As school administrators and educators, fostering an environment that supports these pedagogical practices is key to nurturing the next generation of mathematical thinkers.

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We are excited to share our latest article on the importance of **mathematical models** in education! This piece highlights how enactive, iconic, and symbolic models are essential for helping students transition from concrete understanding to formal reasoning. Iconic models, such as area models and number lines, are crucial in visualizing abstract concepts and fostering deeper learning. The article emphasizes how **progressive formalization** enables students to explore real-world problems and formalize mathematical thinking representations. Take a look and learn how modeling can transform math instruction!


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