DMTI

Math Success

Developing Mathematical Thinking:

Focusing on Models and Language to Enhance Mathematical Understanding

RESEARCH OVERVIEW
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(RESEARCH OVERVIEW)

Introduction

In mathemaÊcs educaÊon, fostering a rich learning environment that promotes diverse problem-solving strategies and emphasizes the structural foundaÊons of mathemaÊcs is crucial for student success. As we navigate curriculum decisions and instructional approaches, it's essenÊal to consider methodologies that encourage students to explore a progression of formalization to solutions while grasping the underlying structures that govern mathemaÊcal concepts.

This research overview focuses on two key elements of effective mathemaÊcs instrucÊon: Encouraging Models and Focusing on Structure through language. With a combinaÊon of theoreÊcal insights and pracÊcal strategies, we provide you with actionable guidance to enhance mathematics instrucÊon within your school.

Encouraging Mathematical Models

Following an iniÊal period of problem-solving and sharing intuiÊve approaches, the subsequent element involves encouraging the exploration, use, and progression of strategies and models. IniÊally, students need opportuniÊes to reflect on their problem-solving approaches and compare them with those of their peers. Subsequently, they should be provided with chances to model problems (Romberg & Kaput, 1999).

Modeling plays a pivotal role in nurturing mathemaÊcal thinking. Students' efforts to model real-world situaÊons form the basis of their understanding. These iniÊal models serve as scaffolding for solving related problems and as a springboard for more formal mathemaÊcal reasoning (Gravemeijer & van Galen, 2003). Cobb (2000) describes this process as a shiḲ in classroom pracÊces, where informal mathemaÊcal acÊviÊes are symbolized and subsequently uÊlized to support more formal mathemaÊcal reasoning across various scenarios (p. 319). Thus, modeling becomes a fundamental aspect of mathematical learning. However, this perspective on models and modeling differs from convenÊonal instructional pracÊces, where models are often used to illustrate expert knowledge, as seen in teaching the regrouping algorithm for subtracÊon using base-10 blocks. Similarly, contextual problems are traditionally introduced only aḲer students have mastered standard algorithmic approaches.

Progressive formalizaÊon guides students through the transition of their models from enacÊve and iconic representaÊons to more formal symbolic representations (Bruner, 1964), avoiding abrupt leaps. This process, encompassing both horizontal and verÊcal mathemaÊzing, focuses on students' uÊlizaÊon of models rather than on teacher-prescribed methods. By embracing aspects of "progressive formalization" and "mathemaÊzing," teachers culÊvate a classroom environment conducive to understanding.

Hence, this element involves cultivaÊng students' comprehension and use of enacÊve, iconic, and symbolic models for understanding and problem-solving (Brunder, 1964; Dolk & Fosnot, 2006; NCTM, 2000; Romberg & Kaput, 1999). When students generate, evaluate, and uÊlize various mathemaÊcal strategies and models, they recognize the mulÊtude of problem-solving avenues and representaÊon methods. Different models illuminate different facets of mathemaÊcs, enriching students' overall understanding of the subject.

Typically, curricula have required students to solve problems in mulÊple ways. However, students oĶen produce solutions that lack disÊnctiveness and fail to promote progressive formalization. In contrast, our module framework offers tasks that encourage students to employ, discuss, and refine both informal and formal models in enactive, iconic, and symbolic forms. Contexts are carefully selected to facilitate logical connecÊons between students' iniÊal informal models and more mathemaÊcally robust ones. For instance, if the area model is a desired iconic representaÊon, students might tackle contextualized problems involving covering flat spaces, such as laying tiles on a floor or using gridlines on a map to determine distances and areas of geographical regions as they then formalize their thinking with symbolic models (Leinwand & Ginsburg, 2007; Watanabe, 2015).

Focusing on Structure with Language

Emphasizing structure enables students to grasp and establish connections among fundamental concepts and specific topics under study (NCTM, 2000; NGA, 2010). Here, structure refers to the immutable elements of mathemaÊcs that persist across grade levels. For instance, concepts like unit, composiÊon, decomposition, iteration, parÊtioning, equivalence, and relaÊonships consÊtute the structural components of the concept of number. Understanding that 28 comprises two units of size 10 and eight units of size one is essenÊal for comprehending place value, just as parÊÊoning one into ten equivalent size units reveals a new unit of one-tenth, which iterates itself tenfold to form one. Consequently, focusing on structure helps students recognize how foundational ideas permeate various grade levels and topics. By highlighÊng connecÊons across different topics, students are liberated from reliance on memorized procedures for individual cases and can instead tackle problems in related contexts.

OĶen, teachers and students perceive mathematics as a progression of increasingly complex procedures and definiÊons throughout the K-12 curriculum. However, fundamental ideas or "structural components" recur consistently across mathemaÊcs, irrespective of grade level.

When instruction neglects the structural underpinnings of mathemaÊcs, students frequently resort to memorized shortcuts or formulas, hindering their ability to solve complex problems or apply mathemaÊcs in novel situaÊons. The DMTI module framework integrates the language of structural components into each lesson's task design and formaÊve assessments, as well as in examples illustrating how students might articulate and criÊque mathemaÊcal models, fostering a deeper understanding of mathemaÊcs.

Conclusion

In conclusion, pursuing excellence in mathemaÊcs educaÊon demands a mulÊfaceted approach that nurtures both flexibility of thought and depth of understanding. By embracing the principles of the progressive formalization of iconic and symbolic **models** and structure with **language**, educators can create classrooms where students are empowered to explore, innovate, and comprehend the parÊculars of mathematics. As school administrators, your support in fostering an environment conducive to such pedagogical pracÊces is pivotal. Together, let us cultivate mathemaÊcal thinkers equipped to thrive in an ever-evolving world.

Brendefur, J.L., Strother, S., Ismail, J. & Krone, K (2011). Developing MathemaĔcal Thinking for Number and Early Algebra Workbook. Developing MathemaĔcal Thinking InsĔtute, Boise, ID.

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