



Developing Mathematical Thinking:  
Taking Students Ideas' Seriously  
DMTI Practice 1

RESEARCH OVERVIEW  
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## Taking Students' Ideas Seriously - DMTI Practice 1

### Introduction

Recent research in mathematics education emphasizes the importance of valuing and building upon students' initial ideas and intuitive understanding. This approach, often referred to as "taking students' ideas seriously," has enhanced conceptual understanding, problem-solving skills, and overall mathematical achievement. This overview examines this approach's theoretical foundations, cognitive processes, and practical implications in mathematics classrooms.

### Theoretical Foundations

Taking students' ideas seriously is grounded in constructivist learning theory and research on how students develop mathematical understanding. Hiebert and Carpenter (1992) argue that "if children possessed internal networks constructed both in and out of school and if they recognized the connections between them, their understanding and performance in both settings would improve." This highlights the importance of connecting students' informal knowledge with formal mathematical concepts. Carpenter's work further emphasizes the value of students' intuitive knowledge: "Children come to school with a great deal of informal or intuitive knowledge of mathematics that can serve as the basis for developing much of the formal mathematics of the primary school curriculum." This suggests that taking students' initial ideas seriously can provide a strong foundation for developing a more sophisticated mathematical understanding.

### Cognitive Processes

When students' ideas are taken seriously in mathematics classrooms, several cognitive processes are engaged:

1. **Schema Formation:** As students articulate and refine their ideas, they develop and modify mental frameworks or schemas that organize mathematical concepts.
2. **Metacognition:** Explaining their thinking engages students' metacognitive processes, promoting reflection on their own understanding and problem-solving strategies.
3. **Elaborative Rehearsal:** Verbalizing mathematical concepts helps move information from working memory to long-term memory, enhancing retention.
4. **Cognitive Conflict:** When students encounter differing viewpoints, it can create cognitive conflict, stimulating the reconciliation of new information with existing schemas.

## Practical Implications

### Eliciting and Valuing Student Ideas

Carpenter and Lehrer argue that for learning with understanding to occur, instruction needs to provide specific opportunities: "For learning with understanding to occur, instruction needs to provide students the opportunity to develop productive relationships, extend and apply their knowledge, reflect about their experiences, articulate what they know, and make knowledge their own." This emphasizes the need for instructional approaches that actively elicit and value student ideas.

### Creating a Supportive Environment

To effectively take students' ideas seriously, teachers must foster a classroom environment where all contributions are respected. This involves:

1. Provide adequate thinking time for students to formulate their thoughts.
2. Using open-ended questions that encourage diverse thinking and approaches.
3. Implementing collaborative strategies like think-pair-share to build confidence in sharing ideas.

### Connecting to Formal Mathematics

Hiebert advocates for teaching practices that promote understanding by focusing on "the inherent structure of the emerging mathematical ideas and addressing students' misconceptions as they arise". This involves helping students connect their informal ideas to more formal mathematical concepts and procedures.

### Impact on Student Learning

Research indicates that taking students' ideas seriously can significantly improve mathematical understanding and achievement. A study by Carpenter et al. (1998) found that when teachers based their instruction on students' thinking, students demonstrated greater problem-solving skills and conceptual understanding compared to control groups. Moreover, this approach has increased student engagement and motivation in mathematics. When students feel their ideas are valued, they are more likely to participate actively in mathematical discussions and take intellectual risks.

### Challenges and Considerations

While the benefits of taking students' ideas seriously are well-documented, implementing this approach can present challenges:

1. **Time Constraints:** Allowing for extended student discussions and idea exploration can be time-consuming within the constraints of a typical school schedule.
2. **Teacher Preparation:** Effectively building on student ideas requires strong content knowledge and pedagogical skills from teachers.
3. **Assessment Alignment:** Traditional assessment methods may not adequately capture the depth of

understanding developed through this approach, necessitating new forms of evaluation.

## Conclusion

Taking students' ideas seriously in mathematics education represents a powerful approach to fostering deep conceptual understanding and problem-solving skills. By valuing students' initial thoughts and building upon their intuitive knowledge, educators can create more engaging and effective learning environments. While challenges exist in implementation, the potential benefits for student learning and mathematical achievement make this approach worthy of serious consideration and further research.

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